Multivalued Fuzzy Mappings and Fixed-Point Results

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Abstract - Here, we have established α -fuzzy fixed point and common α -fuzzy fixed-point theorems for multi-valued fuzzy mapping for complete b-metric space with is an extension and generalization of the results of A. Shahzad with new rational contractive conditions.

Keywords: Multi-valued Mapping (MVM), b-metric space, fixed point, Common fixed point.

I. INTRODUCTION

Fixed point theory has a vital role in development of various field with its applications on Computer Science, Physical Science, Medical Science, Applied Science and areas. In the present scenario with the concept fixed point theory fuzzy locking system, fuzzy machine was been develop. In 1922, Banach [1] prove fixed point theorem for contractive mapping in complete metric space. In 1969, Zadeh [2] introduced the concept of fuzzy set then many author obtain fixed point results for fuzzy mapping. In 1981, Heilpern [4] prove some fixed-point results for contraction mapping. In 1993, The idea of b- metric space Czerwik[5] define and prove fixed point theorem with b-metric space and generalized usual metric space. Afterward many researchers prove many fixed-point theorems for fuzzy contraction mapping using b-metric space. In 2009, Boriceanu[7], In 2016, Joseph[11], In 2017 Jinakul[13] ,Shahzad[12] prove fixed point and common fixed point results for multi-valued mapping in b- metric space. In this paper, we are establishing α -fuzzy fixed point and common α -fuzzy fixed-point theorems for multi-valued fuzzy mapping in complete b-metric space and extending and generalizing the result of A. Shahzad [12] with new rational inequality and we are given for support of them some examples were quoted.

II. PRELIMINARIES

Definition 2.1[1] Let X and Y be a non empty sets. T is said to be Multi-valued mapping from X to Y if T is a Function for X to the Power set Y. we denote a Multi-valued map by $T: X \to 2^Y$

Definition 2.2[1] A fixed point $x_0 \in X$ is said to be fixed point of the Multi-valued mapping T if $x_0 \in Tx_0$.

Example:[4] Every single valued mapping can be Multi-valued mapping. let $f: X \to Y$ be single valued mapping define $T: X \to 2^Y$ by $Tx = \{f(x)\}$. Note that T is a Multi-valued mapping iff for each $x \in X$, $Tx \subseteq Y$. Unless otherwise stated we always assume Tx is non empty for each $x, y \in X$.

Definition 2.3[13] Let (X ,d) be a metric space . A map $T: X \to Y$ is called contraction if there exist $0 \le \rho < 1$ *such that*

$$d(Tx, Ty) \le \rho d(x, y) \; for \; x, y \in X$$

Definition 2.4[1] Let (X, d) be a metric space . A map $T: X \to CB(X)$ is called Multi-valued contraction if there exist $0 \le \rho < 1$ *such that*

$$H(Tx, Ty) \le \rho d(x, y) \; for \; x, y \in X$$

Definition 2.4[6] Let X be any non empty set and $b \ge 1$ be any real number. A function $d: X \times X \to R^+$ is called b-metric if its satisfy the following conditions for all $x, y \in X$

1. $d(x, y) = 0$ *if and only if* $x = y$
2. $d(x, y) = d(y, x)$
3. $d(x, z) \le b[d(x, y) + d(y, z)]$

Then the pair (X, d) is called b-metric space.

Definition 2.4[9] Let (X, d) be a b-metric space. Then the sequence $\{x_n\}$ in X is called Cauchy sequence if and only if for all $\epsilon > 0$ there exists $n(\epsilon) \in N$ such that for each $m, n \ge n(\epsilon)$ we have $d(x_n, x_m) < \epsilon$.

Definition 2.4[6] Let (X, d) be metric space. We define the Housdorff metric space CB(X) induced by d. then

$$H(A, B) = max\{sup_{x \in A} d(x, B), sup_{y \in B} d(A, y)\},$$

For all A, B∈ $CB(X)$, where CB(X) denotes the family of closed and bounded subset of X and $d(x, B) = inf\{d(x, a): a \in B\}$, for all x∈ X.

A Fuzzy set X is a function with domain X and values in [0, 1] , F(X) is the collection of all fuzzy sets in X. If A is a fuzzy set and $x \in X$, then the function value A(x) is called grade of membership of x in X. The α −level set of a fuzzy set A, is denoted by $[A]_\alpha$, and defined as:

$$[A]_\alpha = \{x: A(x) \geq \alpha\}, where\ \alpha \in (0,1]$$

$$[A]_0 = \{x: A(x) > 0\},$$

Let X be any nonempty set and Y be a metric space. A mapping T is called a fuzzy mapping, if T is a mapping from X into F(X). A fuzzy mapping T is a fuzzy subset on $X \times Y$ with membership function T(x)(y). The function T(x)(y) is the grade of membership of y in T(x). For convenience, we denote the α −level set of T(x) by $[Tx]_\alpha$ instead of $[T(x)]_\alpha$ [1].

Definition 2.5([1]) A point $x \in X$ is called α -fuzzy fixed point of a fuzzy mapping $T: X \to F(X)$ if there exist $\alpha \in (0,1]\ such\ that\ x \in [Tx]_\alpha$.

Lemma 2.6 ([1]) Let A and B be a nonempty closed and bounded subsets of a metric space(X, d), if $\alpha \in A\ then$

$$d(a, B) \leq H(A, B).$$

Lemma 2.7 ([1]) Let A and B be a nonempty closed and bounded subsets of a metric space(X, d), and $0 < \alpha \in R\ then\ \ for\ a \in A, there\ exist\ b \in B\ such\ that\ d(a, b) \leq H(A, B) + \epsilon$

In 2017, The following theorem ware proved by A.Shahzad [12] .

Theorem: let (X, d) be a complete b-metric with constant $b \geq 1$ and $T: X \to F(X)$ be a Fuzzy mapping and $for\ x \in X$, $\alpha(x) \in (0,1]$ if satisfying the conditions:

$$\mathrm{H}\big([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}\big) \leq a_1 d\big(x, [Tx]_{\alpha(x)}\big) + a_2 d\big(y, [Ty]_{\alpha(y)}\big) + a_3 d\big(x, [Ty]_{\alpha(y)}\big)$$
$$+ a_4 d\big(y, [Tx]_{\alpha(x)}\big) + a_5 d(x, y) + a_6 \left[\frac{d\big(x, [Tx]_{\alpha(x)}\big) + (1 + d\big(x, [Tx]_{\alpha(x)}\big))}{1 + d(x, y)}\right]$$

For all $x, y \in X\ and\ a_i \geq 0$, $i = 1,2,3..$ with $(ba_1 + a_2 + b(b + 1)a_3 + b(a_5 + a_6)) < 1$

And $\sum_{i=1}^{6} a_i < 1$

Then T has a fixed point.

Now we present our main theorems with new rational contractive conditions.

III. MAIN THEOREM

Theorem (3.1) let (X, d) be a complete b-metric with constant $s \geq 1$ and $U: X \to CB(X)$ be a Multi valued generalized Fuzzy mapping and $x \in X$, $\alpha(x) \in (0,1]$ if satisfying the conditions:

$$\mathrm{H}\big([Ux]_{\alpha(x)}, [Uy]_{\alpha(y)}\big) \leq \propto_1 d(x, y) + \propto_2 d\big(x, [Uy]_{\alpha(y)}\big) + \propto_3 d\big(y, [Uy]_{\alpha(y)}\big)$$
$$+ \propto_4 min\{d\big(x, [Ux]_{\alpha(x)}\big), d\big(y, [Ux]_{\alpha(x)}\big)\} + \propto_5 \left[\frac{d(y, [Ux]_{\alpha(x)}) + d(y, [Uy]_{\alpha(y)})}{1 + d\big(y, [Ux]_{\alpha(x)}\big). d(y, [Uy]_{\alpha(y)})}\right]$$
$$+ \propto_6 d\big(x, [Ux]_{\alpha(x)}\big) \left[\frac{1 + d\big(x, [Ux]_{\alpha(x)}\big) + d\big(y, [Ux]_{\alpha(x)}\big)}{1 + d(x, y)}\right]$$

For all $x, y \in X\ and\ \ \propto_i \geq 0$, $i = 1,2,3.$ with $(\propto_1 + 2s \propto_2 + \propto_3 + \propto_5 + \propto_6) < 1$

And $\sum_{i=1}^{6} \propto_i < 1$

Then U have a $\alpha -$ Fuzzy fixed point.

Proof: Let x_0 be an arbitrary point of X $and\ let\ x_1 \in [Ux_0]_{\alpha(x_0)}$ then by lemma [2.7], we may choose $x_2 \in [Ux_1]_{\alpha(x_1)}$

$$such\ that\ d(x_1, x_2) \leq \mathrm{H}\big([Ux_0]_{\alpha(x_0)}, [Ux_1]_{\alpha(x_1)}\big) + (\propto_1 + s \propto_2 + \propto_6)$$

Now ,

$$d(x_1, x_2) \leq \mathrm{H}\big([Ux_0]_{\alpha(x_0)}, [Ux_1]_{\alpha(x_1)}\big) + (\propto_1 + s \propto_2 + \propto_6)$$

$$\leq \propto_1 d(x_0, x_1) + \propto_2 d\big(x_0, [Ux_1]_{\alpha(x_1)}\big) + \propto_3 d\big(x_1, [Ux_1]_{\alpha(x_1)}\big)$$

$$+ \propto_4 min\{d\big(x_0, [Ux_0]_{\alpha(x_0)}\big), d\big(x_1, [Ux_0]_{\alpha(x_0)}\big)\} + \propto_5 \left[\frac{d(x_1, [Ux_0]_{\alpha(x_0)}) + d(x_1, [Ux_1]_{\alpha(x_1)})}{1 + d\big(x_1, [Ux_0]_{\alpha(x_0)}\big). d(x_1, [Ux_1]_{\alpha(x_1)})}\right]$$

$$+ \propto_6 d\big(x_0, [Ux]_{\alpha(x_0)}\big)\left[\frac{1 + d\big(x_0, [Ux_0]_{\alpha(x_0)}\big) + d\big(x_1, [Ux_0]_{\alpha(x_0)}\big)}{1 + d(x_0, x_1)}\right] + (\propto_1 + s \propto_2 + \propto_6).$$

$$\leq \propto_1 d(x_0, x_1) + \propto_2 d(x_0, x_2) + \propto_3 d(x_1, x_2) + \propto_4 min\{d(x_0, x_1), d(x_1, x_1)\} + \propto_5 \left[\frac{d(x_1, x_1) + d(x_1, x_2)}{1 + d(x_1, x_1). d(x_1, x_2)}\right]$$

$$+ \propto_6 d(x_0, x_1)\left[\frac{1 + d(x_0, x_1) + d(x_1, x_1)}{1 + d(x_0, x_1)}\right] + (\propto_1 + s \propto_2 + \propto_6)$$

$$\leq \propto_1 d(x_0, x_1) + \propto_2 d(x_0, x_2) + \propto_3 d(x_1, x_2) + \propto_5 d(x_1, x_2) + \propto_6 d(x_0, x_1)$$

$$+ (\propto_1 + s \propto_2 + \propto_6)$$

$$\leq \propto_1 d(x_0, x_1) + \propto_2 s[d(x_0, x_1) + d(x_1, x_2)] + \propto_3 d(x_1, x_2) + \propto_5 d(x_1, x_2) + \propto_6 d(x_0, x_1)$$

$$+ (\propto_1 + s \propto_2 + \propto_6)$$

$$[1 - (s \propto_2 + \propto_3 + \propto_5)]\, d(x_1, x_2) \leq (\propto_1 + s \propto_2 + \propto_6) d(x_0, x_1) + (\propto_1 + s \propto_2 + \propto_6)$$

$$d(x_1, x_2) \leq \frac{(\propto_1 + s \propto_2 + \propto_6)}{[(1 - (s \propto_2 + \propto_3 + \propto_5)]} d(x_0, x_1) + \frac{(\propto_1 + s \propto_2 + \propto_6)}{[(1 - (s \propto_2 + \propto_3 + \propto_5)]}$$

Similarly there exist

$x_3 \in [Px_2]_{\alpha(x_2)}$ *such that* $d(x_2, x_3) \leq \mathrm{H}\big([Ux_1]_{\alpha(x_1)}, [Ux_2]_{\alpha(x_2)}\big) + \frac{(\propto_1 + s\propto_2 + \propto_6)^{\,2}}{[(1 - (s\propto_2 + \propto_3 + \propto_5)]}$

Now,

$$d(x_2, x_3) \leq \mathrm{H}\big([Ux_1]_{\alpha(x_1)}, [Ux_2]_{\alpha(x_2)}\big) + \frac{(\propto_1 + s \propto_2 + \propto_6)^{\,2}}{[(1 - (s \propto_2 + \propto_3 + \propto_5)]}$$

$$\leq \propto_1 d(x_1, x_2) + \propto_2 d\big(x_1, [Ux_2]_{\alpha(x_2)}\big) + \propto_3 d\big(x_2, [Ux_2]_{\alpha(x_2)}\big)$$

$$+ \propto_4 min\{d\big(x_1, [Ux_1]_{\alpha(x_1)}\big), d\big(x_2, [Ux_1]_{\alpha(x_1)}\big)\} + \propto_5 \left[\frac{d(x_2, [Ux_1]_{\alpha(x_1)}) + d(x_2, [Ux_2]_{\alpha(x_2)})}{1 + d\big(x_2, [Ux_1]_{\alpha(x_1)}\big). d\big(x_2, [Ux_2]_{\alpha(x_2)}\big)}\right]$$

$$+ \propto_6 d\big(x_1, [Ux_1]_{\alpha(x_1)}\big)\left[\frac{1 + d\big(x_1, [Ux_1]_{\alpha(x_1)}\big) + d\big(x_2, [Ux_1]_{\alpha(x_1)}\big)}{1 + d(x_1, x_2)}\right] + \frac{(\propto_1 + s \propto_2 + \propto_6)^{\,2}}{[(1 - (s \propto_2 + \propto_3 + \propto_5)]}$$

$$\leq \propto_1 d(x_1, x_2) + \propto_2 d(x_1, x_3) + \propto_3 d(x_2, x_3)$$

$$+ \propto_4 min\{d(x_1, x_2), d(x_2, x_2)\} + \propto_5 \left[\frac{d(x_2, x_2) + d(x_2, x_3)}{1 + d(x_2, x_2). d(x_2, x_3)}\right]$$

$$+ \propto_6 d(x_1, x_2)\left[\frac{1 + d(x_1, x_2) + d(x_2, x_2)}{1 + d(x_1, x_2)}\right] + \frac{(\propto_1 + s \propto_2 + \propto_6)^{\,2}}{[(1 - (s \propto_2 + \propto_3 + \propto_5)]}$$

$$\leq \propto_1 d(x_1, x_2) + \propto_2 s[d(x_1, x_2) + d(x_2, x_3)] + \propto_3 d(x_2, x_3) + \propto_5 d(x_2, x_3)$$

$$+ \propto_6 d(x_1, x_2) + \frac{(\propto_1 + s\propto_2 + \propto_6)^{\,2}}{[(1 - (s\propto_2 + \propto_3 + \propto_5)]}$$

$[1-(s \propto_2+\propto_3+\propto_5)]\, d(x_2,x_3) \leq (\propto_1+ s \propto_2+\propto_6)d(x_1 ,x_2) + \frac{(\propto_1+s\propto_2+\propto_6)^2}{[(1-(s\propto_2+\propto_3+\propto_5)]}$

$$d(x_2,x_3) \leq \frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]} d(x_1,x_2) + \frac{(\propto_1+ s \propto_2+\propto_6)^2}{[(1-(s \propto_2+\propto_3+\propto_5)]^2}$$

$$d(x_2,x_3) \leq \left\{\frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]}\right)^2 d(x_0,x_1) + 2\left\{\frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]}\right\}^2$$

$\boldsymbol{d(x_2,x_3) \leq k^2 d(x_0,x_1) + 2k^2}$ **,** where $\boldsymbol{k = \frac{(\propto_1+s\propto_2+\propto_6)}{[(1-(s\propto_2+\propto_3+\propto_5)]}}$
Continuing this process, we obtain a sequence $\{x_n\}$ such that

$d(x_2,x_3) \leq k^2 d(x_0,x_1) + 2k^2$. then

$d(x_n,x_{n+1}) \leq k^n d(x_0,x_1) + nk^n$

Let $u, v > 0\ with\ v > u$

$d(x_u,x_v) \leq s[d(x_u,x_{u+1}) + d(x_{u+1},x_{v)}]$

$d(x_u,x_v) \leq sd(x_u,x_{u+1}) + s^2[d(x_{u+1},x_{u+2}) + d(x_{u+2},x_{v)}]$

$d(x_u,x_v) \leq s[k^u d(x_0,x_1) + uk^u] + s^2[k^{u+1} d(x_0,x_1) + (u+1)k^{u+1}]$

$+ \cdots \ldots + s^{v-u}[k^{v-1} d(x_0,x_1) + (v-1)k^{v-1}]$

$$d(x_u,x_v) \leq sk^u(1 + sk + \cdots \ldots . . + s^{v-u}k^{v-u})d(x_0,x_1) + \sum_{i=u}^{v-1} s^{i-u} i k^i$$

$$d(x_u,x_v) \leq \frac{sk^u}{1-sk} d(x_0,x_1) + \sum_{i=u}^{v-1} s^{i-u} i k^i$$

Since $sk < 1$ by Cauchy root test. $\sum_{i=u}^{v-1} s^{i-u} i k^i$ is convergent. Hence $\{x_n\}$ is Cauchy sequence in X. Since (X,d) be a complete b-metric space the their exist $z \in X\ such\ that\ \lim_{n\to\infty} d(x_n,z) = 0$

.Now we prove z is a Common $\alpha - Fuzzy\ Fixed\ point\ of\ U$. we have

$d(z,[Uz]_{\alpha(z)}) \leq s\big[d(z,x_{n+1}) + d(x_{n+1},[Uz]_{\alpha(z)})\big]$

$d(z,[Uz]_{\alpha(z)}) \leq sd((z,x_{n+1}) + sH([Ux_n]_{\alpha(x_n)},[Uz]_{\alpha(z)})$

$$\begin{aligned} d(z,[Uz]_{\alpha(z)}) \leq\ & sd(z,x_{n+1}) \\ & + s\Big\{\propto_1 d(x_n,z) + \propto_2 d\big(x_n,[Uz]_{\alpha(z)}\big) + \propto_3 d\big(z,[Uz]_{\alpha(z)}\big) \\ & + \propto_4 min\big\{d\big(x_n,[Ux_n]_{\alpha(x_n)}\big), d\big(z,[Ux_n]_{\alpha(x_n)}\big)\big\} + \propto_5 \left[\frac{d\big(z,[Ux_n]_{\alpha(x_n)}\big) + d\big(z,[Uz]_{\alpha(z)}\big)}{1 + d\big(z,[Uz]_{\alpha(z)}\big). d\big(z,[Ux_n]_{\alpha(x_n)}\big)}\right] \\ & + \propto_6 \big[d\big(x_n ,[Ux_n]_{\alpha(x_n)}\big)\left[\frac{1 + d\big(x_n ,[Ux_n]_{\alpha(x_n)}\big) + d\big(z,[Ux_n]_{\alpha(x_n)}\big)}{\big(1 + d(x_n ,z)\big)}\right]\Big]\Big\} \end{aligned}$$

Taking $n \to \infty\ then\ we\ get$

$$d(z,[Uz]_{\alpha(z)}) \leq sd(z,z) + s\left\{\propto_1 d(z,z) + \propto_2 d\big(z,[Uz]_{\alpha(z)}\big) + \propto_3 d\big(z,[Uz]_{\alpha(z)}\big) + \propto_4 min\{d(z,z),d(z,z)\} + \propto_5 \left\{\frac{d(z,z)+d(z,[Uz]_{\alpha(z)})}{1+d\big(z,[Uz]_{\alpha(z)}\big).d(z,z)}\right\} + \propto_6 d(z,z)\left\{\frac{1+d(z,z)+d(z,z)}{1+d(z,z)}\right\}\right\}$$

$$d(z,[Uz]_{\alpha(z)}) \leq s\big[\propto_2 d\big(z,[Uz]_{\alpha(z)}\big) + \propto_3 d\big(z,[Uz]_{\alpha(z)}\big) + \propto_5 d\big(z,[Uz]_{\alpha(z)}\big)\big]$$

$$d(z,[Uz]_{\alpha(z)}) \leq s(\propto_2+\propto_3+\propto_5)d(z,[Uz]_{\alpha(z)}])$$

$$[1-s(\propto_2+\propto_3+\propto_5)]d(z,[Uz]_{\alpha(z)}) \leq 0$$

Which is a Contradiction then we have $d\big(z,[Uz]_{\alpha(z)}\big) = 0$, thus $z = [Uz]_{\alpha(z)}$

Hence z is a $\alpha - fuzzy$ fixed point of U.

Theorem (3.2) let (X,d) be a complete b-metric with constant $s \geq 1$ and $U,P: X \to CB(X)$ be a Two Multi valued generalized fuzzy mapping and $x \in X$, $\alpha_U(x), \alpha_P(y) \in (0,1]$ if satisfying the conditions:

$$H\big([Ux]_{\alpha_U(x)},[Py]_{\alpha_P(y)}\big) \leq \propto_1 d(x,y) + \propto_2 d\big(x,[Py]_{\alpha_P(y)}\big) + \propto_3 d\big(y,[Py]_{\alpha_P(y)}\big) + \propto_4 min\{d\big(x,[Ux]_{\alpha_U(x)}\big), d\big(y,[Ux]_{\alpha_U(x)}\big)\} + \propto_5 \left[\frac{d(y,[Ux]_{\alpha_U(x)}) + d(y,[Py]_{\alpha_P(y)})}{1+d\big(y,[Ux]_{\alpha_U(x)}\big).d\big(y,[Py]_{\alpha_P(y)}\big)}\right] + \propto_6 d\big(x,[Ux]_{\alpha_U(x)}\big)\left[\frac{1+d\big(x,[Ux]_{\alpha_U(x)}\big)+d\big(y,[Ux]_{\alpha_U(x)}\big)}{1+d(x,y)}\right]$$

For all $x,y \in X$ and $\propto_i \geq 0$, $i = 1,2,3.$ with $(\propto_1 + 2s \propto_2 + \propto_3 + \propto_5 + \propto_6) < 1$

And $\sum_{i=1}^{6} \propto_i < 1$

Then U and P have Common $\alpha -$ Fuzzy fixed point.

Proof: Let x_0 be an arbitrary point of X and let $x_1 \in [Ux_0]_{\alpha(x_0)}$ then by lemma [2.7], we may choose $x_2 \in [Px_1]_{\alpha(x_1)}$

$such$ $that$ $d(x_1,x_2) \leq H\big([Ux_0]_{\alpha(x_0)},[Px_1]_{\alpha(x_1)}\big) + (\propto_1 + s \propto_2 + \propto_6)$

Now ,

$$d(x_1,x_2) \leq H\big([Ux_0]_{\alpha(x_0)},[Px_1]_{\alpha(x_1)}\big) + (\propto_1 + s \propto_2 + \propto_4)$$

$$\leq \propto_1 d(x_0,x_1) + \propto_2 d\big(x_0,[Px_1]_{\alpha(x_1)}\big) + \propto_3 d\big(x_1,[Px_1]_{\alpha(x_1)}\big)$$

$$+ \propto_4 min\{d\big(x_0,[Ux_0]_{\alpha(x_0)}\big), d(x_1,[Ux_0]_{\alpha(x_0)}\} + \propto_5 \left[\frac{d\big(x_1,[Ux_0]_{\alpha(x_0)}\big)+d\big(x_1,[Px_1]_{\alpha(x_1)}\big)}{1+d\big(x_1,[Ux_0]_{\alpha(x_0)}\big).d\big(x_1,[Px_1]_{\alpha(x_1)}\big)}\right] + \propto_6 d\big(x_0,[Ux_0]_{\alpha(x_0)}\big)\left[\frac{1+d\big(x_0,[Ux_0]_{\alpha(x_0)}\big)+d\big(x_1,[Ux_0]_{\alpha(x_0)}\big)}{1+d(x_0,x_1)}\right]$$

$$+(\propto_1 + s \propto_2 + \propto_6).$$

$$\leq \propto_1 d(x_0,x_1) + \propto_2 d(x_0,x_2) + \propto_3 d(x_1,x_2) + \propto_4 min\{d(x_0,x_1),d(x_1,x_1)\} + \propto_5 \left[\frac{d(x_1,x_1)+d(x_1,x_2)}{1+d(x_1,x_1).d(x_1,x_2)}\right] + \propto_6 d(x_0,x_1)\left[\frac{1+d(x_0,x_1)+d(x_1,x_1)}{1+d(x_0,x_1)}\right]$$

$+(\propto_1+ s \propto_2+\propto_6).$

$$\leq \propto_1 d(x_0 , x_1) +\propto_2 d(x_0 , x_2) + \propto_3 d(x_1, x_2) +\propto_5 d(x_1, x_2) +\propto_6 d(x_0, x_1)$$

$+(\propto_1+ s \propto_2+\propto_6).$

$$\leq \propto_1 d(x_0 , x_1) +\propto_2 s[d(x_0 , x_1) + d(x_1, x_2)] + \propto_3 d(x_1, x_2) +\propto_5 d(x_1, x_2) +\propto_6 d(x_0, x_1)$$

$+(\propto_1+ s \propto_2+\propto_6).$

$$[1 -(s \propto_2+\propto_3+\propto_5)]\, d(x_1, x_2) \leq (\propto_1+ s \propto_2+\propto_6)d(x_0 , x_1) + (\propto_1+ s \propto_2+\propto_6)$$

$$d(x_1, x_2) \leq \frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]} d(x_0, x_1) + \frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]}$$

Similarly there exist

$x_3 \in [Px_2]_{\alpha(x_2)}$ *such that* $d(x_2, x_3) \leq \mathrm{H}\left([Ux_1]_{\alpha(x_1)}, [Px_2]_{\alpha(x_2)}\right) + \frac{(\propto_1+s\propto_2+\propto_6)^{\,2}}{[(1-(s\propto_2+\propto_3+\propto_5)]}$

Now,

$$d(x_2, x_3) \leq \mathrm{H}\left([Ux_1]_{\alpha(x_1)}, [Px_2]_{\alpha(x_2)}\right) + \frac{(\propto_1+ s \propto_2+\propto_6)^{\,2}}{[(1-(s \propto_2+\propto_3+\propto_5)]}$$

$$\leq \propto_1 d(x_1 , x_2) +\propto_2 d\left(x_1 , [Px_2]_{\alpha(x_2)}\right) +\propto_3 d\left(x_2, [Px_2]_{\alpha(x_2)}\right)$$

$$+ \propto_4 min\left\{d\left(x_1, [Ux_1]_{\alpha(x_1)}\right), d\left(x_2, [Ux_1]_{\alpha(x_1)}\right)\right\} +\propto_5 \left[\frac{d\left(x_2, [Ux_1]_{\alpha(x_1)}\right) + d\left(x_2, [Px_2]_{\alpha(x_2)}\right)}{1 + d\left(x_2, [Ux_1]_{\alpha(x_1)}\right). d\left(x_2, [Px_2]_{\alpha(x_2)}\right)}\right]$$

$$+\propto_6 d\left(x_1, [Ux_1]_{\alpha(x_1)}\right)\left[\frac{1 + d\left(x_1, [Ux_1]_{\alpha(x_1)}\right) + d\left(x_2, [Ux_1]_{\alpha(x_1)}\right)}{1 + d(x_1, x_2)}\right] + \frac{(\propto_1+ s \propto_2+\propto_6)^{\,2}}{[(1-(s \propto_2+\propto_3+\propto_5)]}$$

$$\leq \propto_1 d(x_1 , x_2) +\propto_2 d(x_1 , x_3) +\propto_3 d(x_2, x_3)$$

$$+ \propto_4 min\{d(x_1, x_2), d(x_2, x_2)\} +\propto_5 \left[\frac{d(x_2, x_2) + d(x_2, x_3)}{1 + d(x_2, x_2). d(x_2 , x_3)}\right]$$

$$+\propto_6 d(x_1, x_2)\left[\frac{1 + d(x_1, x_2) + d(x_2, x_2)}{1 + d(x_1, x_2)}\right] + \frac{(\propto_1+ s \propto_2+\propto_6)^{\,2}}{[(1-(s \propto_2+\propto_3+\propto_5)]}$$

$$\leq \propto_1 d(x_1 , x_2) +\propto_2 s[d(x_1 , x_2) + d(x_2, x_3)] +\propto_3 d(x_2, x_3) +\propto_5 d(x_2, x_3)$$

$+\propto_6 d(x_1, x_2)+ \frac{(\propto_1+s\propto_2+\propto_6)^{\,2}}{[(1-(s\propto_2+\propto_3+\propto_5)]}$

$$[1 -(s \propto_2+\propto_3+\propto_5)]\, d(x_2, x_3) \leq (\propto_1+ s \propto_2+\propto_6)d(x_1 , x_2) + \frac{(\propto_1+s\propto_2+\propto_6)^2}{[(1-(s\propto_2+\propto_3+\propto_5)]}$$

$$d(x_2, x_3) \leq \frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]} d(x_1, x_2) + \frac{(\propto_1+ s \propto_2+\propto_6)^{\,2}}{[(1-(s \propto_2+\propto_3+\propto_5)]^2}$$

$$d(x_2, x_3) \leq \left\{\frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]}\right)^2 d(x_0, x_1) + 2\left\{\frac{(\propto_1+ s \propto_2+\propto_6)}{[(1-(s \propto_2+\propto_3+\propto_5)]}\right\}^2$$

$\boldsymbol{d(x_2, x_3) \leq k^2 d(x_0, x_1) + 2k^2}$ **,** where $\boldsymbol{k = \frac{(\propto_1 + s\propto_2 + \propto_6)}{[(1-(s\propto_2 + \propto_3 + \propto_5)]}}$

Continuing this process, we obtain a sequence $\{x_n\}$ such that

$d(x_2, x_3) \leq k^2 d(x_0, x_1) + 2k^2$. then

$d(x_n, x_{n+1}) \leq k^n d(x_0, x_1) + nk^n$

Let $u, v > 0$ *with* $v > u$

$$d(x_u, x_v) \leq s[d(x_u, x_{u+1}) + d(x_{u+1}, x_{v)}]$$

$$d(x_u, x_v) \leq sd(x_u, x_{u+1}) + s^2[d(x_{u+1}, x_{u+2}) + d(x_{u+2}, x_{v)}]$$

$$d(x_u, x_v) \leq s[k^u d(x_0, x_1) + uk^u] + s^2[k^{u+1} d(x_0, x_1) + (u+1)k^{u+1}]$$

$$+ \cdots \ldots + s^{v-u}[k^{v-1} d(x_0, x_1) + (v-1)k^{v-1}]$$

$$d(x_u, x_v) \leq sk^u(1 + sk + \cdots \ldots \ldots + s^{v-u}k^{v-u})d(x_0, x_1) + \sum_{i=u}^{v-1} s^{i-u} i k^i$$

$$d(x_u, x_v) \leq \frac{sk^u}{1 - sk} d(x_0, x_1) + \sum_{i=u}^{v-1} s^{i-u} i k^i$$

Since $sk < 1$ by Cauchy root test. $\sum_{i=u}^{v-1} s^{i-u} i k^i$ is convergent. Hence $\{x_n\}$ is Cauchy sequence in X. Since (X, d) be a complete b-metric space the their exist $z \in X$ *such that* $\lim_{n\to\infty} d(x_n, z) = 0$

.Now we prove z is a Common $\alpha - \mathit{Fuzzy\ Fixed\ point\ of\ U}$. we have

$$d(z, [Uz]_{\alpha(z)}) \leq s\left[d(z, x_{2n+1}) + d(x_{2n+1}, [Uz]_{\alpha(z)})\right]$$

$$d(z, [Uz]_{\alpha(z)}) \leq sd((z, x_{2n+1}) + sH([Ux_{2n}]_{\alpha(x_{2n})}, [Uz]_{\alpha(z)})$$

$$\begin{aligned} d(z, [Uz]_{\alpha(z)}) \leq\ & sd(z, x_{2n+1}) \\ & + s\Bigg\{\propto_1 d(x_{2n}, z) + \propto_2 d\big(x_{2n}, [Uz]_{\alpha(z)}\big) + \propto_3 d\big(z, [Uz]_{\alpha(z)}\big) \\ & + \propto_4 \min\{d\big(x_{2n}, [Ux_{2n}]_{\alpha(x_{2n})}\big), d\big(z, [Ux_{2n}]_{\alpha(x_{2n})}\big)\} + \propto_5 \left[\frac{d(z, [Ux_{2n}]_{\alpha(x_{2n})}) + d(z, [Uz]_{\alpha(z)})}{1 + d\big(z, [Ux_{2n}]_{\alpha(x_{2n})}\big). d(z, [Uz]_{\alpha(z)})}\right] \\ & + \propto_6 d\big(x_{2n}, [Ux_{2n}]_{\alpha(x_{2n})}\big)\left[\frac{1 + d\big(x_{2n}, [Ux_{2n}]_{\alpha(x)}\big) + d\big(z, [Ux_{2n}]_{\alpha(x_{2n})}\big)}{1 + d(x_{2n}, z)}\right]\Bigg\} \end{aligned}$$

Taking $n \to \infty$ *then we get*

$$\begin{aligned} d(z, [Uz]_{\alpha(z)}) \leq\ & sd(z, z) \\ & + s\Bigg\{\propto_1 d(z, z) + \propto_2 d\big(z, [Uz]_{\alpha(z)}\big) + \propto_3 d\big(z, [Uz]_{\alpha(z)}\big) + \propto_4 \min\{d(z, z), d(z, z)\} \\ & + \propto_5 \left[\frac{d(z, z) + d(z, [Uz]_{\alpha(z)})}{1 + d(z, z). d(z, [Uz]_{\alpha(z)})}\right] + \propto_6 d(z, z)\left[\frac{1 + d(z, z) + d(z, z)}{1 + d(z, z)}\right]\Bigg\} \end{aligned}$$

$$d(z, [Uz]_{\alpha(z)}) \leq s\left[\propto_2 d\big(z, [Uz]_{\alpha(z)}\big) + \propto_3 d\big(z, [Uz]_{\alpha(z)}\big) + \propto_5 d\big(z, [Uz]_{\alpha(z)}\big)\right]$$

$$d(z, [Uz]_{\alpha(z)}) \leq s(\propto_2 + \propto_3 + \propto_5) d(z, [Uz]_{\alpha(z)}])$$

$$[1 - s(\propto_2 + \propto_3 + \propto_5)] d(z, [Uz]_{\alpha(z)}) \leq 0$$

Which is a Contradiction then we have $d(z,[Uz]_{\alpha(z)}) = 0$, thus $z = [Uz]_{\alpha(z)}$

$$d(z,[Pz]_{\alpha(z)}) \leq s[d(z,x_{2n+1}) + d(x_{2n+1},[Pz]_{\alpha(z)})]$$

$$d(z,[Pz]_{\alpha(z)}) \leq sd((z,x_{2n+1}) + sH([Px_{2n}]_{\alpha(x_{2n})},[Pz]_{\alpha(z)})$$

$$\begin{aligned} d(z,[Pz]_{\alpha(z)}) \leq\ & sd(z,x_{2n+1}) \\ & + s\Big\{\propto_1 d(x_{2n},z) + \propto_2 d(x_{2n},[Pz]_{\alpha(z)}) + \propto_3 d(z,[Pz]_{\alpha(z)}) \\ & + \propto_4 min[d(x_{2n},[Px_{2n}]_{\alpha(x_{2n})}), d(z,[Px_{2n}]_{\alpha(x_{2n})})] + \propto_5 \left[\frac{d(z,[Px_{2n}]_{\alpha(x_{2n})}) + d(z,[Pz]_{\alpha(z)})}{1 + d(z,[Px_{2n}]_{\alpha(x_{2n})}). d(z,[Pz]_{\alpha(z)})}\right] \\ & + \propto_6 d(x_{2n},[Px_{2n}]_{\alpha(x_{2n})}) \left[\frac{1 + d(x_{2n},[Px_{2n}]_{\alpha(x)}) + d(z,[Px_{2n}]_{\alpha(x_{2n})})}{1 + d(x_{2n},z)}\right]\Big\} \end{aligned}$$

Taking $n \to \infty$ *then we get*

$$d(z,[Pz]_{\alpha(z)}) \leq sd(z,z) + s\left\{\begin{array}{c} \propto_1 d(z,z) + \propto_2 d(z,[Pz]_{\alpha(z)}) + \propto_3 d(z,[Pz]_{\alpha(z)}) \\ + \propto_4 min[d(z,z), d(z,z)] + \propto_5 \left\{\frac{d(z,z) + d(z,[Pz]_{\alpha(z)})}{1 + d(z,z). d(z,z)}\right\} \\ + \propto_6 d(z,z)\left[\frac{1 + d(z,z) + d(z,z)}{1 + d(z,z)}\right] \end{array}\right\}$$

$$d(z,[Pz]_{\alpha(z)}) \leq s\{\propto_2 d(z,[Pz]_{\alpha(z)}) + \propto_3 d(z,[Pz]_{\alpha(z)}) + \propto_5 d(z,[Pz]_{\alpha(z)})\}$$

$$d(z,[Pz]_{\alpha(z)}) \leq s(\propto_2 + \propto_3 + \propto_5) d(z,[Pz]_{\alpha(z)}$$

$$[1 - s(\propto_2 + \propto_3 + \propto_5)] d(z,[Pz]_{\alpha(z)}) \leq 0$$

Which is a Contradiction Then we have $d(z,[Pz]_{\alpha(z)}) = 0$, thus $z = [Pz]_{\alpha(z)}$

Hence z *is a* Common $\alpha - fuzzy\ fixed\ point\ of\ U\ and\ P$.

Theorem (3.3) let (X, d) be a complete b-metric with constant $s \geq 1$ and $U: X \to CB(X)$ be a Multi valued generalized Fuzzy mapping and $x \in X$, $\alpha(x) \in (0,1]$ if satisfying the conditions:

$$\begin{aligned} H([Ux]_{\alpha(x)},[Uy]_{\alpha(y)}) \leq & \propto_1 d(x,y) + \propto_2 d(x,[Uy]_{\alpha(y)}) + \propto_3 d(y,[Uy]_{\alpha(y)}) \\ & + \propto_4 d(x,[Ux]_{\alpha(x)}) + \propto_5 d(x,[Ux]_{\alpha(x)}) \left[\frac{1 + d(x,[Ux]_{\alpha(x)}) + d(y,[Ux]_{\alpha(x)})}{1 + d(x,y)}\right] \\ & + \propto_6 \left[\frac{d(y,[Ux]_{\alpha(x)}) + d(y,[Uy]_{\alpha(y)})}{1 + d(y,[Ux]_{\alpha(x)}). d(y,[Uy]_{\alpha(y)})}\right] \end{aligned}$$

For all $x, y \in X$ *and* $\propto_i \geq 0$, $i = 1,2,3..$ with $(\propto_1 + 2s \propto_2 + \propto_3 + \propto_4 + \propto_5 + \propto_6) < 1$

And $\sum_{i=1}^{6} \propto_i < 1$

Then U have a $\alpha -$ Fuzzy fixed point.

Proof: Similar proof as theorem (3.1)

Example: Let $X = [0,1]$ *and* $U(x,y) = |x - y| \ \forall x,y \in X$, then (X,d) be a complete b-metric space.

Define a fuzzy mapping $U: X \to F(X)\ by\ U(x)(t) = \begin{cases} 1, 0 \leq t \leq \frac{x}{8} \\ \frac{1}{2}, \frac{x}{8} < t \leq \frac{x}{4} \\ \frac{1}{4}, \frac{x}{4} < t \leq \frac{x}{2} \\ 0, \frac{x}{2} < t \leq 1 \end{cases}$

For all $x \in X, there\ exists\ \alpha(x) = 1, such\ that\ [Ux]_{\alpha(x)} = \left[0, \frac{x}{8}\right]. then$

$$\mathrm{H}\left([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}\right) \leq \frac{1}{4} \left| \left(x - \frac{x}{8}\right) \right| + \frac{1}{8} \left| \left(y - \frac{y}{8}\right) \right| + \frac{1}{16} \left| \left(x - \frac{y}{8}\right) \right| + \frac{1}{32} \left| \left(y - \frac{x}{8}\right) \right| + \frac{1}{64} \left| (x - y) \right| + \frac{1}{128} \left[\frac{|x - x/4| + (1 + |x - x/4|}{1 + |x - y|} \right]$$

Therefore, $0 \in X$ is the fixed point of U.

IV. CONCLUSION

By using b-metric space, many authors have fixed point results for self-mapping. In this paper by using b-metric space we prove the existence and uniqueness of fixed-point results for Multivalued fuzzy mapping. Our result extent and generalized the result of A. Shahzad with new rational expression.

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