Fixed Point Theorems via E.A like Property in Fuzzy Metric Space

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Abstract: In this paper, we prove the existence and uniqueness of a common fixed point in which the pair of maps satisfy the E.A. like property. This generalizes the result of Bhingare et al. (2013) and can be useful for researchers seeking to establish a common fixed point for rational expression. We also define a new class of function ψ.

Keywords: Fuzzy metric space, Common Fixed point, Common E.A like property, weakly compatible mapping

I. INTRODUCTION

In 1975, Kromosil and Michalek initially introduced the idea of fuzzy metric space [1]. The modified Continuous t-norm technique was first presented in 1994 by George and Veeramani [7]. Many writers and researchers used a number of innovative concepts—compatible mapping, weak compatible mapping, R-weakly Computing mapping, and CLR-property, among others—to generate new insights on fuzzy metric space in diverse ways. In 2002, Sushil Sharma [10] established several common fixed-point theorems and defined the property E.A. for the first time as a novel concept in fuzzy metric spaces under strict contractive conditions.

Many writers and scholars created new in 2013, K. Wadhawa et al. introduced the original idea of the E.A. like property in fuzzy metric space [4]. This characteristic is essential for guaranteeing that one does not require range subspace proximity, mapping continuity, or the completeness of the entire space. Numerous domains, such as communication, applied sciences, stability theory, control theory, neural network theory, image processing, and medical sciences.

This work presents the proof of a few common fixed-point theorems for new combination six and for four mappings through weakly compatible and common E.A.-like properties in a fuzzy metric space.

II. PRELIMINARIES

Definition 2.1[16] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norm if * satisfying the condition:

(1) * is commutative and associative.

(2) * is continuous.

(3) $a * 1 = a\ , \forall a \in [0,1]$

(4) $a * b\ \leq c * d\ , whenever\ a \leq c\ and\ b\ \leq d\ for\ \forall\ a, b, c, d\ \in [0,1]$

Definition 2.2[15] A 3-tuple $(X, M, *)$ is said to be Fuzzy Metric space if X is an arbitrary set, * is a continuous t-norm and M is a Fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: $\forall\ x, y, z\ \in X\ s, t > 0$

(1) $M(x, y, z) > 0$

(2) $M(x, y, t) = 1\ for\ all\ t > 0\ iff\ x = y.$

(3) $M(x, y, t) = M(z, x, t)$

(4) $M(x, y, t_1) * M(z, z, t_2) \leq M(x, z, t_1 + t_2)$

$$\forall x, y, z\ \in X\ and\ t_1, t_2, > 0$$

(5) $X(x, y, *): [0, \infty] \rightarrow [0,1]$ is left continuous.

Example 2.3(Induced fuzzy metric space [15]): Let (X, d) be a metric space. Denote $a * b = ab$ for all $a, b \in [0,1]$ and let M be a fuzzy set on $X^2 \times [0, \infty)$ defined as follows:

$$m(x, y, t) = \frac{t}{t + d(x, y)}$$

Definition 2.4[4] Two self mapping A and B on a fuzzy mapping $(X, M, *)$ are said to be compatible if $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1\ for\ all\ t > 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = u\ ,\ for\ all\ u \in X$

Definition 2.5[13] Two self-mapping A and B on a fuzzy mapping $(X, M,*)$ are said to be a weakly compatible if they commute at their coincidence point that is for $\forall\, u \in X\, , Au = Bu$ *implies that* $ABu = BAu$ *for all* $t > 0$

Definition 2.6[4] Two self-mapping A and B on a fuzzy mapping $(X, M,*)$ are said to be Satisfy E.A property if their exist a sequence $\{x_n\}$ in Xsuch that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = u\,,\ for\ all\ u \in X$$

Definition 2.7[4] Let A, B,S and T be a self-mapping of a Fuzzy Metric space $(X, M,*)$ then (A,S) and (B,T) said to satisfy Common E.A like property if their exist two sequence $\{x_n\}$ *and* $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} \mathrm{T} y_n = z$, Where $z \in S(X) \cap T(X)\ or\ z \in A(X) \cap B(X)$

Lemma 2.8[5] $(X, M,*)$ is non decreasing function *for all* $x, y, z \in X$

The following definition and result are define Mishra[5].

Lemma 2.9[4] Let$(X, M,*)$ be a Fuzzy Metricspace if their exist $h \in (0,1)$ *such that*

$$M(x, y, ht) \geq M(x, y, t)\ then\ x = y\ and\ , h \in (0,1), t > 0\ for\ all\ x, y \in X$$

Main Result

Theorem 3.1: Let A, B, S, T, P *and* Q be a self mapping of a fuzzy metric space $(X, M,*)$ satisfying the following conditions:

(i) $P(X) \subseteq ST(X), Q(X) \subseteq AB(X)$;

(ii) $AB = BA, ST = TS, PB = BP, QT = TQ$;

(iii) Either $AB\ or\ P$ is continuous;

(iv) (P, AB) *and* (Q, ST) are weakly compatible.

(v) Pair (P, AB) *and* (Q, ST)follows E.A Property.

(vi) $\forall x, y, z \in x$ and $t > 0$.

$$M(Px, Qy, t) \geq p\left\{Max\begin{pmatrix} M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), \\ M(Px, STy, t) \end{pmatrix}\right\}$$

Where $r: [0,1] \to [0,1]$ is continuous function such that $p(t) > t$ for each $0 < t < 1, p(0) = 0$ and $p(1) = 1$ then A, B, S, T, P and Q have a unique common fixed point.

Proof: Since (P, AB) and (Q, ST) satisfying common E.A like properties then there exist two sequences $\{x_n\}$ *and* $\{y_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} AB x_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = z$.

Where $, z \in AB(X) \cap ST(X)\ or\ z \in P(X) \cap Q(X)$.

Let $z \in AB(X) \cap ST(X)$ and $\lim_{n\to\infty} Px_n = z \in AB(X)\ then\ z = ABu$, where $x \in X$.

To Prove, $Pu = ABu$

Put $x = u\ and\ y = y_n$ in inequality (vi)

$$M(Pu, Qy_n, t) \geq p\left\{Max\begin{pmatrix} M(ABu, STy_n, t), M(Pu, ABu, t), M(Qy_n, STy_n, t), \\ M(Pu, STy_n, t) \end{pmatrix}\right\}$$

$$M(Pu, z, t) \geq p\left\{Max\begin{pmatrix} M(Pu, z, t), M(Pu, Pu, t), M(z, z, t), \\ M(Pu, z, t) \end{pmatrix}\right\}$$

$$M(Pu, z, t) \geq p\left\{Max\begin{pmatrix} M(Pu, z, t), 1,1, \\ M(Pu, z, t) \end{pmatrix}\right\}$$

$$M(Pu, z, t) \geq p(1) = 1$$

This implies that $Pu = z\ i.e\ Pu = z = ABu$

Since (P, AB) is weakly compatible then $Pz = PABu = ABPu = ABz$

Now, we claim that $Pz = z$

Put $x = z$ and $y = y_n$ in (vi)

$$M(Pz, Qy_n, t) \geq p\left\{Max\begin{pmatrix} M(ABz, STy_n, t), M(Pz, ABz, t), M(Qy_n, STy_n, t), \\ M(Pz, STy_n, t) \end{pmatrix}\right\}$$

$$M(Pz, z, t) \geq p\left\{Max\begin{pmatrix} M(Pz, z, t), M(Pz, Pz, t), M(z, z, t), \\ M(Pz, z, t) \end{pmatrix}\right\}$$

$$M(Pz, z, t) \geq p\left\{Max\begin{pmatrix} M(Pz, z, t), 1, 1, \\ M(Pz, z, t) \end{pmatrix}\right\}$$

$M(Pz, z, t) \geq r(1) = 1$

This implies that $Pz = z$ $i.e$ $Pz = z = ABz$

Again put $x = Bz$ and $y = y_n$ in (vi)

$$M(PBz, Qy_n, t) \geq p\left\{Max\begin{pmatrix} M(ABBz, STy_n, t), M(PBz, ABBz, t), M(Qy_n, STy_n, t), \\ M(PBz, STy_n, t) \end{pmatrix}\right\}$$

Taking limit $n \to \infty$ and also, $AB = BA, PB = BP$

So we get, $P(Bz) = B(Pz) = Bz$ and $AB(Bz) = BA(Bz) = B(ABz) = Bz$

$$M(Bz, z, t) \geq p\left\{Max\begin{pmatrix} M(Bz, z, t), M(Bz, Bz, t), M(z, z, t), \\ M(PBz, z, t) \end{pmatrix}\right\}$$

$$M(Bz, z, t) \geq p\left\{Max\begin{pmatrix} M(Bz, z, t), 1, 1, \\ M(Bz, z, t) \end{pmatrix}\right\}$$

$M(Bz, z, t) \geq p(1) = 1$

This implies that $Bz = z$ $then$ $ABz = z \Rightarrow Az = z$

Therefore $Az = Bz = Pz = z$.

Again, Let $z \in ST(X)$ for some $v \in X$ $then$ $z = STv$

Now to Prove, $Qv = STv$

Put $x = x_n$ and $y = v$ in inequality (vi)

$$M(Px_n, Qv, t) \geq p\left\{Max\begin{pmatrix} M(ABx_n, STv, t), M(Px_n, ABx_n, t), M(Qv, STv, t), \\ M(Px_n, STv, t) \end{pmatrix}\right\}$$

$$M(z, Qv, t) \geq p\left\{Max\begin{pmatrix} M(z, z, t), M(z, z, t), M(Qv, z, t), \\ M(z, z, t) \end{pmatrix}\right\}$$

$$M(z, Qv, t) \geq p\left\{Max\begin{pmatrix} 1, 1, M(Qv, z, t), \\ 1 \end{pmatrix}\right\}$$

$M(z, Qv, t) \geq p(1) = 1$

This implies that $Qv = z \Rightarrow Qv = zSTv$

Since (Q, ST) is weakly compatible then $Qz = QSTv = STQv = STz$

Now to Prove $Qz = z$ then

Put $x = x_n$ and $y = z$ in inequality (vi)

$$M(Px_n, Qz, t) \geq p\left\{Max\begin{pmatrix} M(ABx_n, STz, t), M(Px_n, ABx_n, t), M(Qz, STz, t), \\ M(Px_n, STz, t) \end{pmatrix}\right\}$$

$$M(z, Qz, t) \geq p\left\{Max\begin{pmatrix} M(z, Qz, t), M(z, z, t), M(Qz, Qz, t), \\ M(z, Qz, t) \end{pmatrix}\right\}$$

$$M(z,Qz,t) \geq p\left\{Max\begin{pmatrix}M(z,Qz,t),1,1,\\ M(z,Qz,t)\end{pmatrix}\right\}$$

$M(z,Qz,t) \geq p(1) = 1$

This implies that $Qz = z \Rightarrow Qz = z = STz$

Put $x = x_n$ and $y = Tz$ in (vi)

$$M(Px_n,QTz,t) \geq p\left\{Max\begin{pmatrix}M(ABx_n,STTz,t),M(Px_n,ABx_n,t),M(QTz,STTz,t),\\ M(Px_n,STTz,t)\end{pmatrix}\right\}$$

Since $Q(Tz) = T(Qz) = Tz$

$ST(Tz) = TS(Tz) = T(STz) = Tz$

$$M(z,Tz,t) \geq p\left\{Max\begin{pmatrix}M(z,Tz,t),M(z,z,t),M(Tz,Tz,t),\\ M(z,Tz,t)\end{pmatrix}\right\}$$

$$M(z,Tz,t) \geq r\left\{Max\begin{pmatrix}M(z,Tz,t),1,1,\\ M(z,Tz,t)\end{pmatrix}\right\}$$

$M(z,Tz,t) \geq p(1) = 1$

This implies that $Tz = z \Rightarrow Tz = z = STz = Sz$

Therefore $Tz = Qz = Sz = z$.

Hence $Az = Bz = Tz = Sz = Qz = Pz = z$.

Hence A, B, S, T, P and T have a unique Common Fixed Point.

Uniqueness: Suppose z_1 and z_2 are two common fixed points of A, B, S, T, P and $T$with $z_1 \neq z_2$ then from (vi)

$$M(Pz_1,Qz_2,t) \geq p\left\{Max\begin{pmatrix}M(ABz_1,STz_2,t),M(Pz_1,ABz_1,t),M(Qz_2,STz_2,t),\\ M(Pz_2,STz_2,t)\end{pmatrix}\right\}$$

$$M(z_1,z_2,t) \geq p\left\{Max\begin{pmatrix}M(z_1,z_2,t),M(z_1,z_1,t),M(z_2,z_2,t),\\ M(z_2,z_2,t)\end{pmatrix}\right\}$$

$M(z_1,z_2,t) \geq p\{Max(M(z_1,z_2,t))\}$

$$M(z_1,z_2,t) \geq p(M(z_1,z_2,t)) > (M(z_1,z_2,t))$$

This is contradiction.

Hence, $z_1 = z_2$.

Remark * *let us we define new class of* ψ *as follows.*

Let ψ *be the class of all mapping* $\psi : [0,1] \to [0,1]$ *such that*

(a) ψ *is non-decreasing and* $\lim_{n\to\infty} \psi^n(b) = 1, \forall b \in (0,1]$;

(b) $\psi(b) > b, \forall b \in (0,1)$;

(c) $\psi(1) = 1$;

Example : *Define* $\psi: [0,1] \to [0,1]$ *by* $\psi(r) = \frac{2r}{r+1}, \forall p \in [0,1]$,

$$\psi^2(r) = \frac{4r}{3r+1}, \psi^3(r) = \frac{8r}{7r+1} \dots\dots, \psi^n(r) = \frac{2^n r}{(2^n-1)r+1}, \forall p \in [0,1].$$

$$\lim_{n\to\infty} \psi^n(r) = \frac{2^n r}{(2^n-1)r+1} = 1, \forall r \in [0,1]$$

$$Clearly, \psi(r) > r \; and \; \psi(1) = 1, \forall r \in [0,1]$$

Theorem 3.2: Let P, G, N and H be a self mapping of a Fuzzy Metric space $(X, M, *)$ with $a * b = \min(a, b)$, satisfying the following conditions:

(i) (P,N) *and* (G,H) Satisfying common E.A like properties.

(ii) (P,N) *and* (G,H) are weakly compatible.

(iii) $M(Px,Gy,t) \geq \psi \begin{Bmatrix} \varpi(M(Nx,Hy,t)), \varpi(M(Nx,Gy,2t)), \varpi(M(Px,Hy,t)), \\ \varpi(M(Nx,Px,t)), \end{Bmatrix}$

$\forall\ x,y \in X\ and\ t > 0$

$then\ P,G,N\ and\ H\ have\ a\ Common\ Fixed\ Point.$

Proof: Since (P,N) *and* (G,H) Satisfying common E.A like properties then there exist two sequences $\{x_n\}$ *and* $\{y_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} Gy_n = \lim_{n\to\infty} Hy_n = z.$

Where , $z \in N(X) \cap T(X)\ or\ z \in P(X) \cap B(X)$

Let $z \in N(X) \cap H(X)$ and $\lim_{n\to\infty} Px_n = z \in N(X)\ then\ z = Nu$, where $x \in X$

To Prove, $Pu = Nu$

Put $x = u\ and\ y = y_n$ in (iii)

$$M(Pu,Gy_n,ht) \geq \phi \begin{Bmatrix} \varpi(M(Nu,Hy_n,t)), \varpi(M(Nu,Gy_n,t)), \varpi(M(Pu,Hy_n,t)), \\ \varpi(M(Nu,Pu,t)) \end{Bmatrix}$$

$$M(Pu,z,ht) \geq \phi \begin{Bmatrix} \varpi(M(z,z,t)), \varpi(M(z,z,t)), \varpi(M(Pu,z,t)), \\ \varpi(M(z,Pu,t)) \end{Bmatrix}$$

$$M(Pu,z,ht) \geq \phi \begin{Bmatrix} \varpi(1), \varpi(1), \varpi(M(Pu,z,t)), \\ \varpi(M(z,Pu,t)) \end{Bmatrix}$$

$by\ using\ remark *$ Properties then we get

$$M(Pu,z,ht) \geq \phi\{\,1\,,1\,,M(Pu,z,t), M(z,Pu,t), 1\,\}$$

$$M(Pu,z,kt) \geq\ M(Pu,z,,t)$$

By lemma 2.9, we get $Pu = z\ i.e\ Pu = z = Nu$

Since (P,N) *is* weakly compatible then $Pz = PNu = NPu = Nz$

Again, $\lim_{n\to\infty} Gy_n = z \in H(X)\ then\ z = Hv$

Now to Prove, $Hv = Gv$

Put $x = x_n\ and\ y = v\ in\ (iii)$

$$M(Px_n,Gv,ht) \geq \phi \begin{Bmatrix} \varpi(M(Nx_n,Hv,t)), \varpi(M(Nx_n,Hv,t)), \varpi(M(Nx_n,Gv,t)) \\ ,\varpi(M(Nx_n,Px_n,t)) \end{Bmatrix}$$

$$M(z,Gv,ht) \geq \phi \begin{Bmatrix} \varpi(M(z,z,t)), \varpi(M(z,Gv,t)), \varpi(M(z,z,t)), \\ \varpi(M(z,z,t)) \end{Bmatrix}$$

$$M(z,Gv,ht) \geq \phi \begin{Bmatrix} \varpi(1), \varpi(M(z,Gv,t)), \varpi(1), \\ \varpi(1) \end{Bmatrix}$$

$by\ using\ remark *$ Properties then we get

$$M(z,Gv,kt) \geq \phi\{1, M(z,Gv,t), 1,1\,\}$$

$$\mathfrak{M}(z,Gv,kt) \geq M(z,Uv,t)$$

By lemma 2.9, we get $z = Gv\ i.e\ Gv = z = Hv$

Since (U,H) is weakly compatible then $Gz = GHv = HGv = Hz$

Now to Prove $Pz = z$ then

Put $x = z\ and\ y = y_n$ in (iii)

$$M(Pz, By_n, ht) \geq \phi \begin{Bmatrix} \varpi\big(M(Nz, Hy_n, t)\big), \varpi\big(M(Nz, Gy_n, t)\big), \varpi\big(M(Pz, Hy_n, t)\big), \\ \varpi\big(M(Nz, Pz, t)\big) \end{Bmatrix}$$

$$M(Pz, z, ht) \geq \phi \begin{Bmatrix} \varpi\big(M(Pz, z, t)\big), \varpi\big(M(Pz, z, t)\big), \varpi\big(M(Pz, z, t)\big), \\ \varpi\big(M(Pz, Pz, t)\big) \end{Bmatrix}$$

$$M(Pz, z, ht) \geq \phi\{ \varpi\big(M(Pz, z, t)\big), \varpi\big(M(Pz, z, t)\big), \varpi\big(M(Pz, z, t)\big), \varpi(1)\}$$

$by\ using\ remark *$ Properties then we get

$$M(Pz, z, ht) \geq \phi\{ M(Pz, z, t), M(Pz, z, t), M(Pz, z, t), 1,1\}$$

$M(Pz, z, ht) \geq M(Pz, z, t)$

By lemma 2.9, we get $Pz = z$ i.e $Pz = z = Nz$

Now to Prove $Gz = z$

Put $x = x_n\ and\ y = z$ in (iii)

$$M(Px_n, Gz, kt) \geq \phi \begin{Bmatrix} \varpi\big(M(Nx_n, Tz, t)\big), \varpi\big(M(Nx_n, Gz, t)\big), \varpi\big(M(Px_n, Hz, t)\big), \\ \varpi\big(M(Nx_n, Px_n, \mathrm{t})\big) \end{Bmatrix}$$

$$M(z, Gz, ht) \geq \phi \begin{Bmatrix} \varpi\big(M(z, Gz, t)\big), \varpi\big(M(z, Gz, t)\big), \varpi\big(M(z, Gz, t)\big), \\ \varpi\big(M(z, z, t)\big) \end{Bmatrix}$$

$$M(z, Gz, ht) \geq \phi\{ \varpi\big(M(z, Gz, t)\big), \varpi\big(M(z, Gz, t)\big), \varpi\big(M(z, Gz, t)\big), \varpi(1)\}$$

$by\ using\ remark *$ Properties then we get

$$M(z, Bv, ht) \geq \phi\{M(z, Bz, t), M(z, Bz, t), M(z, Bz, t), 1\}$$

$M(z, Gz, ht) \geq M(z, Gz, t)$

By lemma 2.9, we get $Gz = z\ i.e\ Gz = z = Hz$.

Hence, $Pz = Gz = Nz = Hz = z$.

Hence $P, G, N\ and\ H\ have\ a\ Common\ Fixed\ Point$.

Uniqueness: Suppose z_1 and z_2 are two common fixed point of $P, G, N\ and\ H$ with $z_1 \neq z_2$ then from (3.2)

$$M(Pz_1, Gz_2, ht) \geq \phi \begin{Bmatrix} \varpi\big(M(Nz_1, Hz_2, t)\big), \varpi\big(M(Nz_1, Gz_2, t)\big), \varpi\big(M(Pz_1, Hz_2, t)\big), \\ \varpi\big(M(Nz_1, Pz_1, t)\big) \end{Bmatrix}$$

$$M(z_1, z_2, ht) \geq \phi \begin{Bmatrix} \varpi\big(M(z_1, z_2, t)\big), \varpi\big(M(z_1, z_2, t)\big), \varpi\big(M(z_1, z_2, t)\big), \\ \varpi\big(M(z_1, z_2, t)\big) \end{Bmatrix}$$

$$M(z_1, z_2, ht) \geq \phi\{ \varpi\big(M(z_1, z_2, t)\big), \varpi\big(M(z_1, z_2, t)\big), \varpi\big(M(z_1, z_2, t)\big), \varpi(1)\}$$

$$M(z_1, z_2, ht) \geq \phi\{M(z_1, z_2, t), M(z_1, z_2, t) M(z_1, z_2, t), 1\}$$

By definition of Implicit function, we get, $M(z_1, z_2, ht) \geq \phi\{M(z_1, z_2, t)\}$

By lemma 2.9, we get. $z_1 = z_2$.

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